

# Parameterized total cross sections for pion production in nuclear collisions

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## Abstract

Total inclusive cross sections for neutral and charged pion production in proton-nucleus and nucleus-nucleus reactions have been calculated and compared to experiment. Nucleon-nucleon theoretical cross sections have been scaled up to nuclear collisions using a scaling factor similar to  $(A_P A_T)^{2/3}$ , where  $A_P$  and  $A_T$  are the nucleon numbers of the projectile and target nuclei. Variations in the power of this scaling factor have been studied and a good fit to experiment is obtained with a small modification of the power. Theoretical cross sections are written in a form that is very suitable for immediate input into transport codes.

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## 1 Introduction

Radiation transport codes [1] often use arithmetic parameterizations of cross sections, in order that the transport code is able to run quickly. We have previously developed parameterizations of pion production cross sections in proton-proton reactions [2]. It is worthwhile to extend these to proton-nucleus and nucleus-nucleus reactions for use in heavy-ion transport codes, which are needed for space radiation protection [1] and other applications. Pion production is important in several ways. After neutral pions are produced they decay electromagnetically to high energy photons which initiate an electromagnetic cascade consisting of other high energy photons, electrons and positrons. The high energy photons, resulting from nucleon-nucleon collisions in the interstellar medium, are easily seen by gamma ray telescopes. The high energy electrons can penetrate space suits and therefore are important during EVA (extra-vehicular activity). Charged

pions can produce ionization before decaying and are therefore a source of radiation by themselves. High energy muons result from their decay and it is well known that these muons can reach the surface of the Earth due to relativistic time dilation. Such muons can also penetrate space suits. The peak of the cosmic ray spectrum occurs near the GeV region, which we refer to as intermediate energy. This is the region of interest for space radiation applications. The aim of the present work is to provide simple parameterizations of total pion production cross sections in nuclear collisions, which can be used as input for heavy-ion transport codes.

## 2 Nucleon-nucleon cross sections

The total inclusive cross section for pion production in proton-proton collisions has been parameterized as [2]

$$\sigma_{pp \rightarrow \pi^0 X} = \left( 0.007 + 0.1 \frac{\ln(T_{\text{lab}})}{T_{\text{lab}}} + \frac{0.3}{T_{\text{lab}}^2} \right)^{-1} \quad (1)$$

$$\sigma_{pp \rightarrow \pi^+ X} = \left( 0.00717 + 0.0652 \frac{\ln(T_{\text{lab}})}{T_{\text{lab}}} + \frac{0.162}{T_{\text{lab}}^2} \right)^{-1} \quad (2)$$

$$\sigma_{pp \rightarrow \pi^- X} = \left( 0.00456 + \frac{0.0846}{T_{\text{lab}}^{0.5}} + \frac{0.577}{T_{\text{lab}}^{1.5}} \right)^{-1} \quad (3)$$

where  $T_{\text{lab}}$  should be specified in GeV to give  $\sigma$  in mb.

The trouble with only having proton-proton (pp) cross sections is that nuclei also contain neutrons, and so one needs cross sections for pion production from neutron-neutron (nn) and neutron-proton (np) collisions. The *exclusive* reactions for single pion production are listed below.

$$pp \rightarrow pp\pi^0 \quad [\times 2] \quad (4)$$

$$\rightarrow pn\pi^+ \quad [\times 2] \quad (5)$$

and

$$nn \rightarrow nn\pi^0 \quad [\times 2] \quad (6)$$

$$\rightarrow np\pi^- \quad [\times 2] \quad (7)$$

and

$$pn \rightarrow pn\pi^0 \quad [\times 2] \quad (8)$$

$$\rightarrow nn\pi^+ \quad [\times 2] \quad (9)$$

$$\rightarrow pp\pi^- \quad [\times 2] \quad (10)$$

The number in square brackets after some reactions indicates that the reaction can proceed in a number of different ways and therefore the number of particles produced needs to be multiplied by the number in square brackets. For example the reaction  $pp \rightarrow pn\pi^+$  can also proceed as  $pp \rightarrow np\pi^+$ , with the pion being produced from the other nucleon. Exclusive reactions for double pion production in terms of initial states are

$$pp \rightarrow pp\pi^0\pi^0 \quad (11)$$

$$\rightarrow pp\pi^+\pi^- \quad [\times 2] \quad (12)$$

$$\rightarrow pn\pi^+\pi^0 \quad [\times 2] \quad (13)$$

$$\rightarrow nn\pi^+\pi^+ \quad (14)$$

and

$$nn \rightarrow nn\pi^0\pi^0 \quad (15)$$

$$\rightarrow nn\pi^+\pi^- \quad [\times 2] \quad (16)$$

$$\rightarrow pp\pi^-\pi^- \quad (17)$$

$$\rightarrow np\pi^-\pi^0 \quad [\times 2] \quad (18)$$

and

$$pn \rightarrow np\pi^0\pi^0 \quad (19)$$

$$\rightarrow np\pi^+\pi^- \quad [\times 2] \quad (20)$$

$$\rightarrow nn\pi^+\pi^0 \quad [\times 2] \quad (21)$$

$$\rightarrow pp\pi^-\pi^0 \quad [\times 2] \quad (22)$$

By considering the reactions with  $pp$  in the initial state, we see that the ratio of the numbers of pions is

$$\pi^+ : \pi^- : \pi^0 = 8 : 2 : 6 = 4 : 1 : 3 \quad (23)$$

Thus at *low energy* (i.e. around the two pion threshold) we expect the  $pp$  cross sections to be in this ratio, namely

$$\sigma_{pp \rightarrow \pi^+ X} : \sigma_{pp \rightarrow \pi^- X} : \sigma_{pp \rightarrow \pi^0 X} = 4 : 1 : 3 \quad (\text{low energy}) \quad (24)$$

We therefore expect that if we divide the  $\sigma_{pp \rightarrow \pi^+ X}$  by 4 and  $\sigma_{pp \rightarrow \pi^0 X}$  by 3 then the three pion cross sections should be roughly the same at low energy. If one plots the cross sections this is seen to be approximately true for low energy.

Thus we conclude that the  $\pi^+$  to  $\pi^-$  ratio for pp reactions at low energy is given by

$$\frac{\sigma_{pp \rightarrow \pi^+ X}}{\sigma_{pp \rightarrow \pi^- X}} = 4 \quad (\text{low energy}) \quad (25)$$

A similar analysis leads us to conclude that the  $\pi^+$  to  $\pi^-$  ratio for nn reactions at low energy is given by

$$\frac{\sigma_{nn \rightarrow \pi^+ X}}{\sigma_{nn \rightarrow \pi^- X}} = \frac{1}{4} \quad (\text{low energy}) \quad (26)$$

and for pn reactions

$$\frac{\sigma_{pn \rightarrow \pi^+ X}}{\sigma_{pn \rightarrow \pi^- X}} = 1 \quad (27)$$

### 3 Nucleus-Nucleus cross sections

For nucleus-nucleus collisions, denote the probability of a proton-proton reaction as

$$\text{Prob(pp)} = \frac{Z_P Z_T}{A_P A_T} \quad (28)$$

where  $Z$  and  $A$  are the proton and nucleon numbers of the Projectile and Target. The probability of a neutron-neutron reaction is

$$\text{Prob(nn)} = \frac{A_P - Z_P}{A_P} \frac{A_T - Z_T}{A_T} \quad (29)$$

For nucleus-nucleus collisions at low energy we therefore expect the  $\pi^+$  to  $\pi^-$  ratio to be

$$\frac{\sigma_{AA \rightarrow \pi^+ X}}{\sigma_{AA \rightarrow \pi^- X}} = \text{Prob(pp)} \times 4 + \text{Prob(nn)} \times \frac{1}{4} \quad (\text{low energy}) \quad (30)$$

$$= \left( \frac{Z_P Z_T}{A_P A_T} \times 4 \right) + \left( \frac{A_P - Z_P}{A_P} \frac{A_T - Z_T}{A_T} \times \frac{1}{4} \right) \quad (\text{low energy}) \quad (31)$$

Generalising to higher energy therefore gives

$$\frac{\sigma_{AA \rightarrow \pi^+ X}}{\sigma_{AA \rightarrow \pi^- X}} = \left( \frac{Z_P Z_T}{A_P A_T} \frac{\sigma_{pp \rightarrow \pi^+ X}}{\sigma_{pp \rightarrow \pi^- X}} \right) + \left( \frac{A_P - Z_P}{A_P} \frac{A_T - Z_T}{A_T} \frac{\sigma_{pp \rightarrow \pi^- X}}{\sigma_{pp \rightarrow \pi^+ X}} \right) \quad (32)$$

However this argument only gives us the *ratio*. We need a model to get one of the cross sections. We first note that  $\pi^+$  production can be well described using

$$\sigma_{AA \rightarrow \pi^+ X} \approx (A_P A_T)^{2/3} \sigma_{pp \rightarrow \pi^+ X} \quad (33)$$

In order to obtain  $\sigma_{AA \rightarrow \pi^- X}$  we proceed as follows. Substitute (33) into (32) to obtain

$$\sigma_{AA \rightarrow \pi^- X} = \frac{(A_P A_T)^{2/3} \sigma_{pp \rightarrow \pi^+ X}}{\left( \frac{Z_P Z_T}{A_P A_T} \frac{\sigma_{pp \rightarrow \pi^+ X}}{\sigma_{pp \rightarrow \pi^- X}} \right) + \left( \frac{A_P - Z_P}{A_P} \frac{A_T - Z_T}{A_T} \frac{\sigma_{pp \rightarrow \pi^- X}}{\sigma_{pp \rightarrow \pi^+ X}} \right)} \quad (34)$$

Equations (33) and (34) represent formulas for charged pion production from nucleus-nucleus collisions. However it will be seen below that somewhat better fits to data are obtained by slightly changing the 2/3 power appearing in (33) and (34). Therefore the final formulas for pion production in proton-nucleus and nucleus-nucleus collisions are

$$\sigma_{AA \rightarrow \pi^+ X} = (A_P A_T)^{N_{\pm}} \sigma_{pp \rightarrow \pi^+ X} \quad (35)$$

$$\sigma_{AA \rightarrow \pi^- X} = \frac{(A_P A_T)^{N_{\pm}} \sigma_{pp \rightarrow \pi^+ X}}{\left( \frac{Z_P}{A_P} \frac{Z_T}{A_T} \frac{\sigma_{pp \rightarrow \pi^+ X}}{\sigma_{pp \rightarrow \pi^- X}} \right) + \left( \frac{A_P - Z_P}{A_P} \frac{A_T - Z_T}{A_T} \frac{\sigma_{pp \rightarrow \pi^- X}}{\sigma_{pp \rightarrow \pi^+ X}} \right)} \quad (36)$$

$$\sigma_{AA \rightarrow \pi^0 X} = (A_P A_T)^{N_0} \sigma_{pp \rightarrow \pi^0 X} \quad (37)$$

where equations (1, 2, 3) are used to obtain the elementary pion cross sections. We have found that the best fit to the data is obtained with the values

$$N_{\pm} = 2.2/3 \quad (38)$$

$$N_0 = 2.4/3 \quad (39)$$

Comparisons have been made between the theory developed above and the data from References [3] and [4]. The comparisons can be seen in Figures 1 - 8. It is evident that agreement between theory and experiment is good.

## 4 Conclusions

The present work has investigated neutral and charged pion production in proton-nucleus and nucleus-nucleus collisions at the intermediate energy (i.e. in the GeV region) relevant to space radiation. Total cross sections for inclusive processes have been calculated and compared to experiment. The parameterizations developed in equations (35, 36, 37, 38, 39) are in good agreement with experiment. For charged pion production in nucleus-nucleus reactions the theory overlaps all error bars from 0.4 to 2.1 AGeV (Figs. 1 - 5). For neutral pion production in nucleus-nucleus reactions at 1 AGeV there are only 3 data points and the agreement is satisfactory (Fig. 6). For proton-nucleus reactions, only  $\pi^-$  data is available. The agreement between theory and experiment is satisfactory Figs. 7 - 8. The equations developed herein are in a form that is suitable for immediate input into heavy-ion transport codes. The formalism is expected to be valid for total cross sections in the GeV region.

## Acknowledgements

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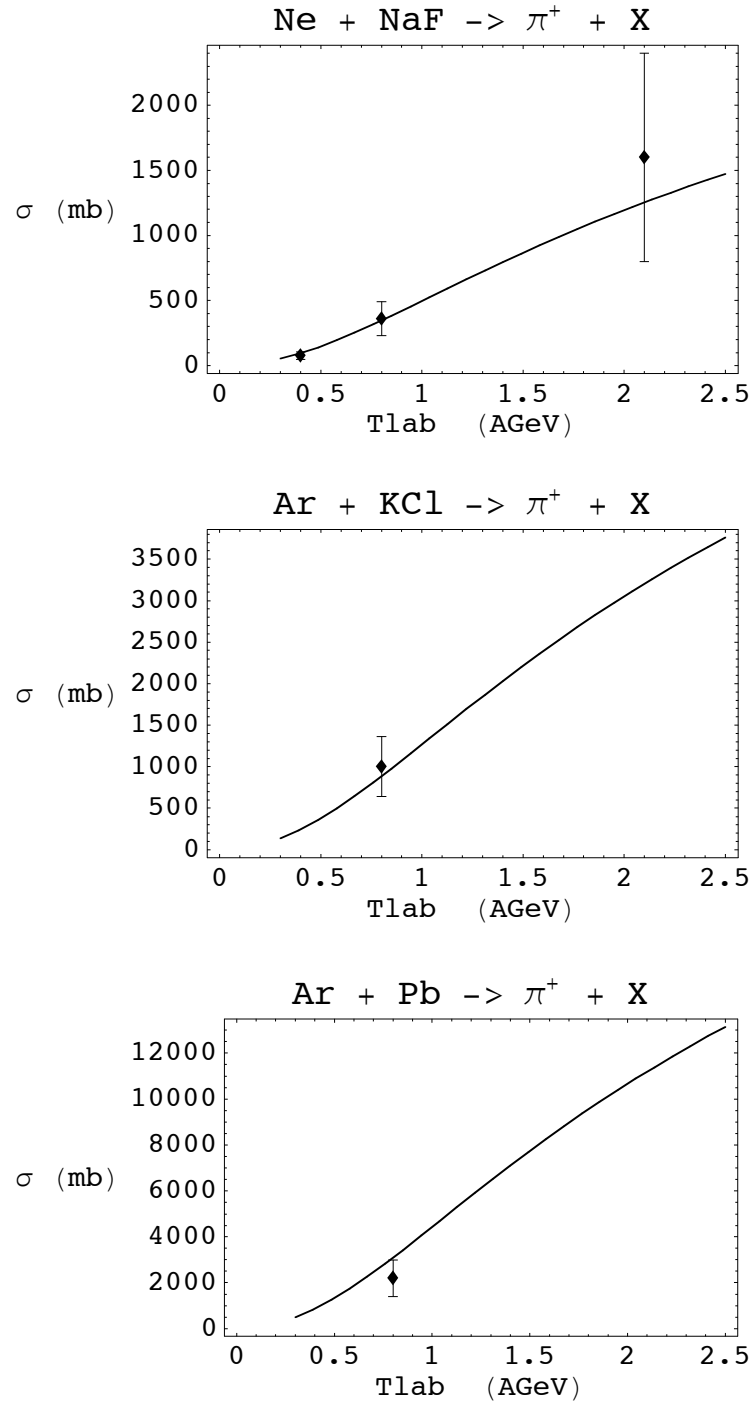


Figure 1. Theory versus experiment for  $\pi^+$  production in nucleus-nucleus collisions. The line corresponds to equation (35). Data are from reference [3].

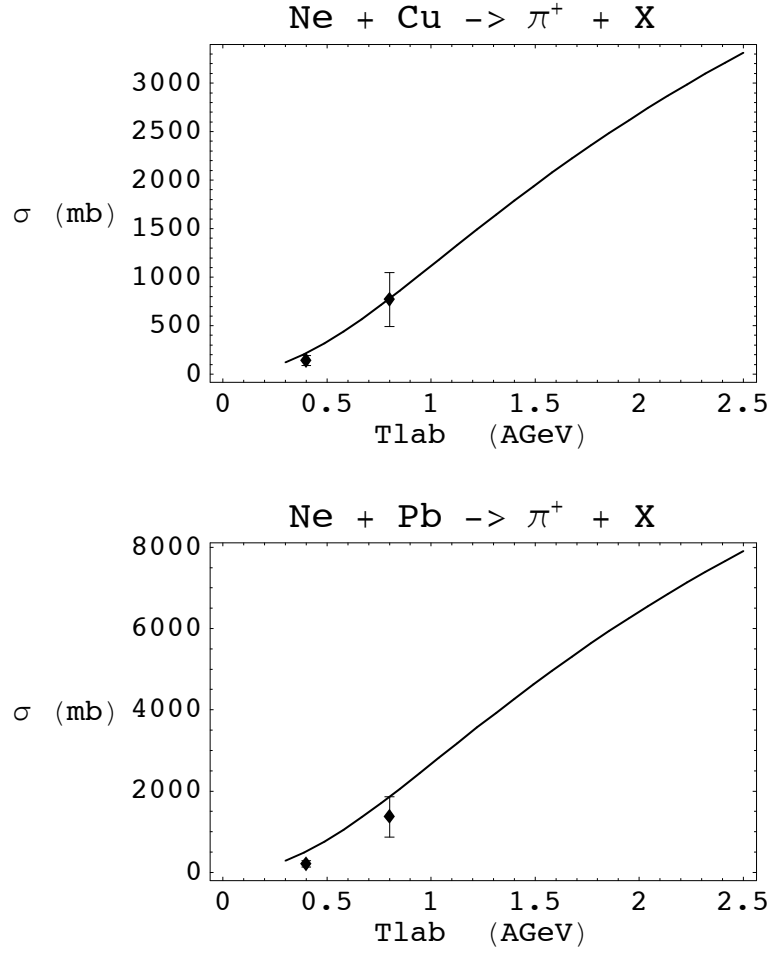


Figure 2. Theory versus experiment for  $\pi^+$  production in nucleus-nucleus collisions. The line corresponds to equation (35). Data are from reference [3].

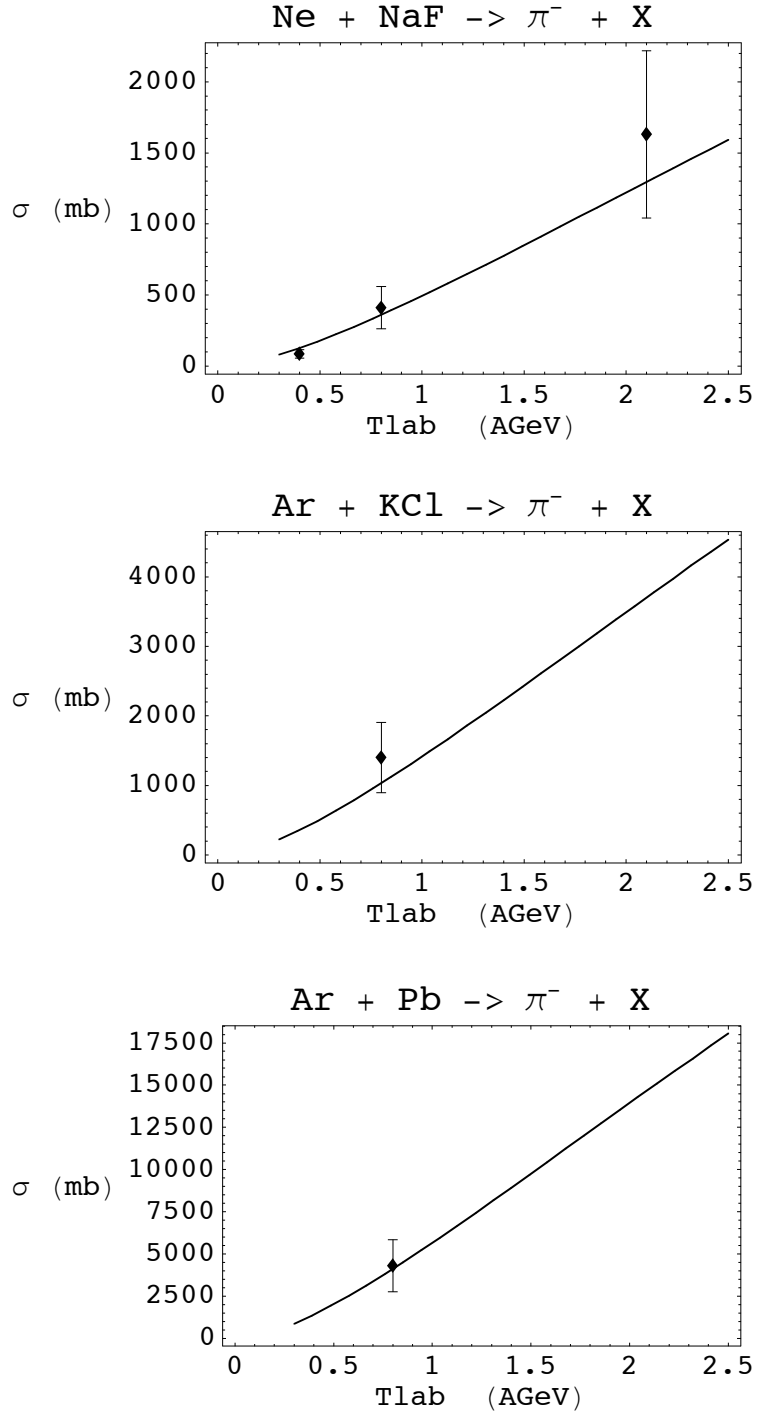


Figure 3. Theory versus experiment for  $\pi^-$  production in nucleus-nucleus collisions. The line corresponds to equation (36). Data are from reference [3].



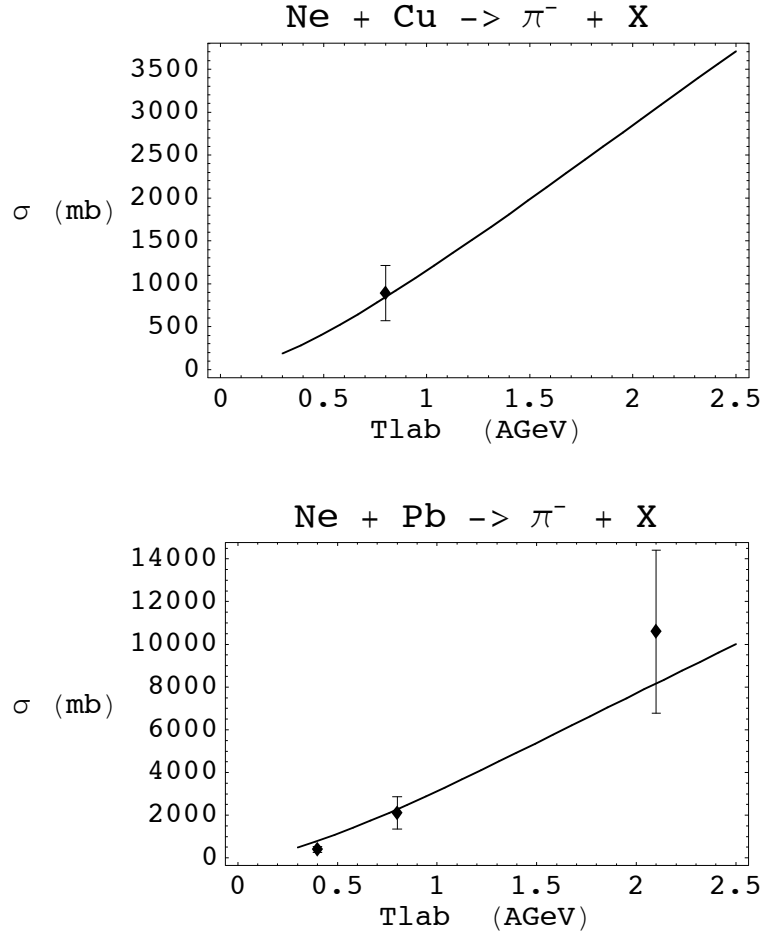


Figure 4. Theory versus experiment for  $\pi^-$  production in nucleus-nucleus collisions. The line corresponds to equation (36). Data are from reference [3].

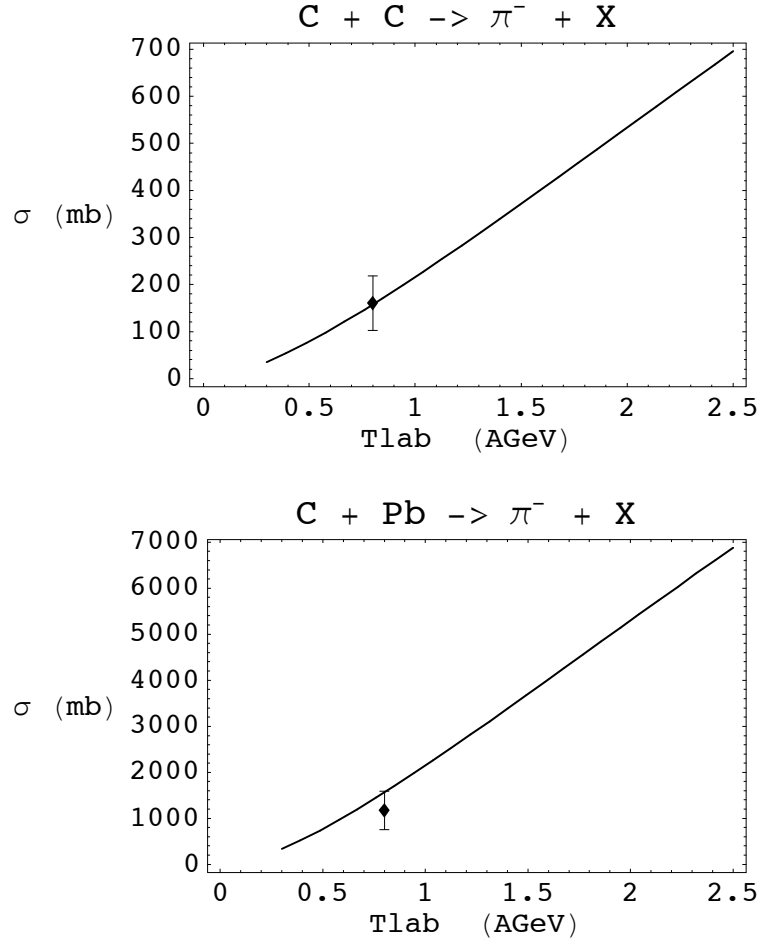


Figure 5. Theory versus experiment for  $\pi^-$  production in nucleus-nucleus collisions. The line corresponds to equation (36). Data are from reference [3].

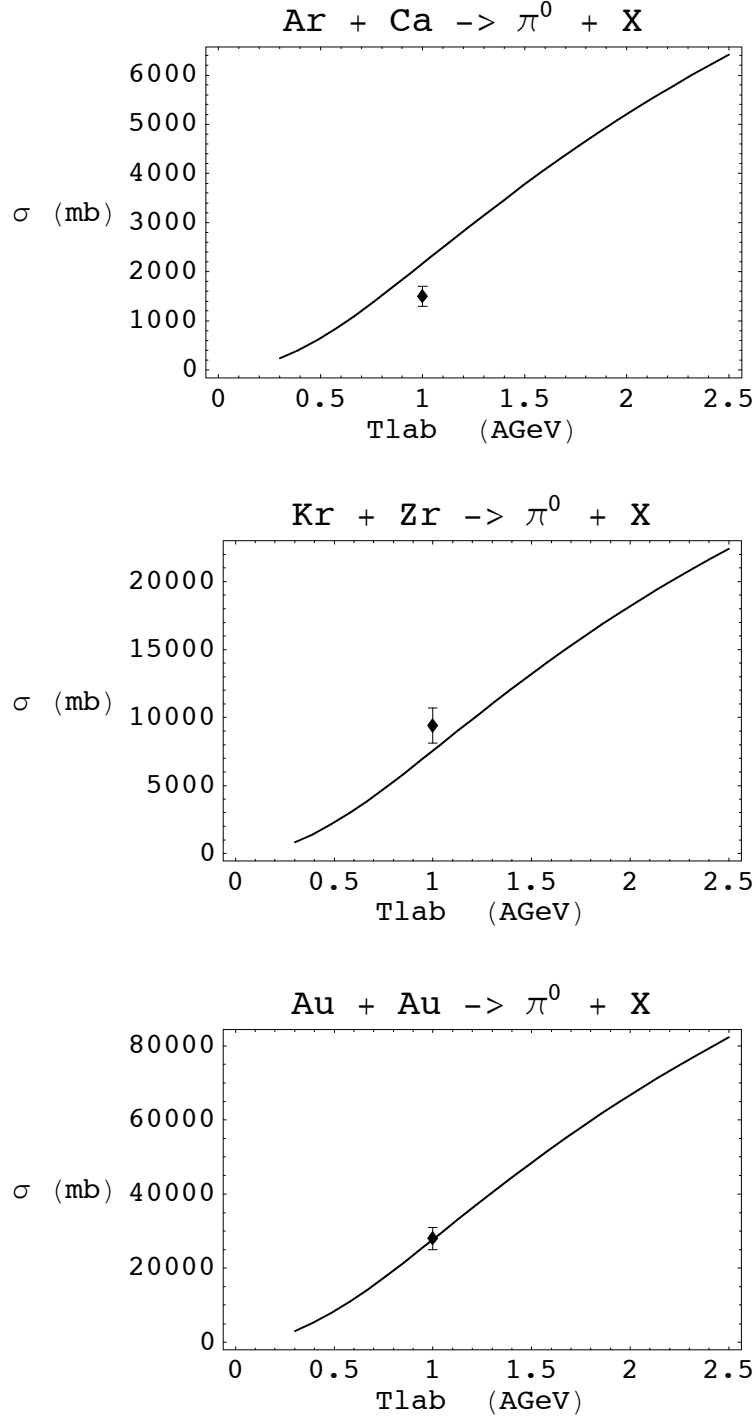


Figure 6. Theory versus experiment for  $\pi^0$  production in nucleus-nucleus collisions. The line corresponds to equation (37). Data are from reference [4].

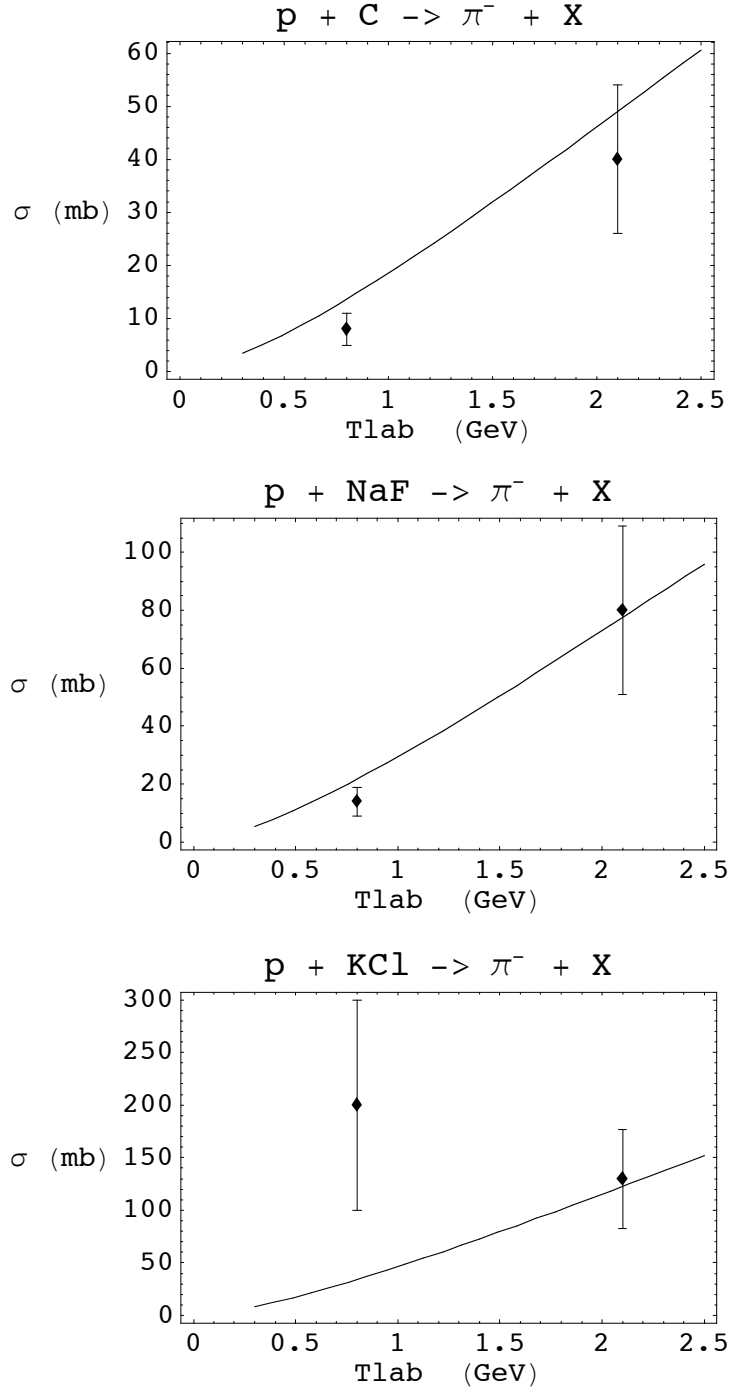


Figure 7. Theory versus experiment for  $\pi^-$  production in proton-nucleus collisions. The line corresponds to equation (36). Data are from reference [3].

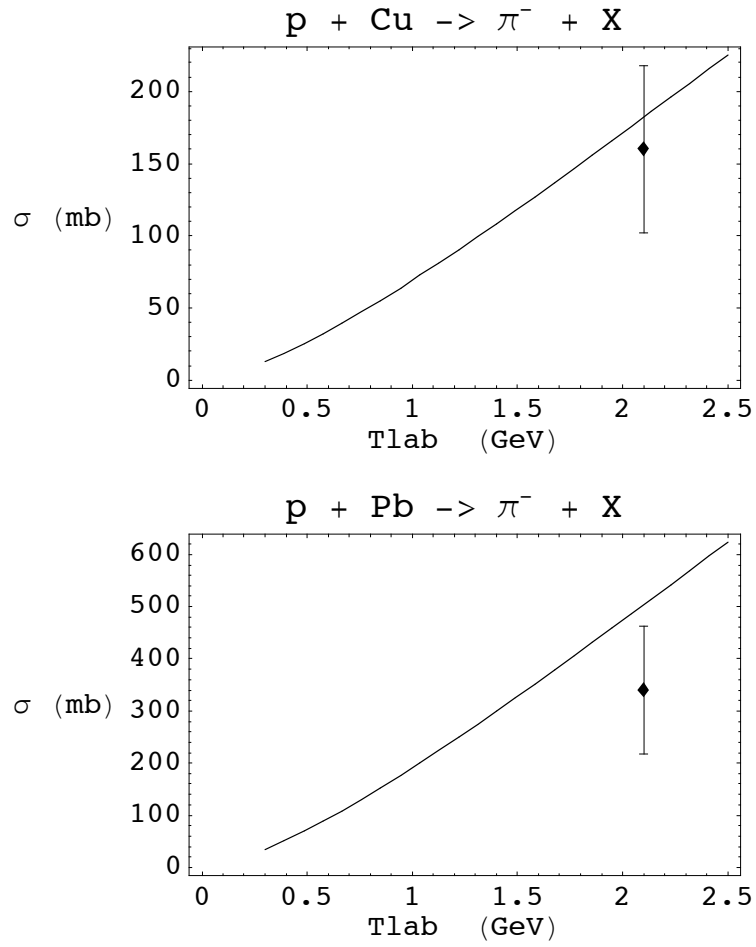


Figure 8. Theory versus experiment for  $\pi^-$  production in proton-nucleus collisions. The line corresponds to equation (36). Data are from reference [3].

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